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The open string pair-production rate enhancement by a magnetic flux

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Abstract: We extend the amplitude calculations of [1] to exhaust the remaining cases for which one set of D_p branes carrying a flux (electric or magnetic) is placed parallel at separation to the other set carrying also a flux but with the two fluxes sharing at most one common field-strength index. We then find that the basic structure of amplitudes remains the same when the two fluxes share at least one common index but it is more general when the two fluxes share no common index. We discuss various properties of the amplitudes such as the large separation limit, the onset of various instabilities and the open string pair production. In particular, when one flux is electric and weak and the other is magnetic and fixed, we find that the open string pair production rate is greatly enhanced by the presence of this magnetic flux when the two fluxes share no common field-strength index and this rate becomes significant when the separation is on the order of string scale.

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Contents

1 Introduction

One type of non-perturbative solitonic objects in superstring theories (see, for example, [2]) is nowadays called D-branes [3]. The lowest order stringy interaction between two such parallel Dp-branes separated by a distance can be computed either as an open string one-loop annulus diagram with one end of the open string located at one D-brane and the other end at the other D-brane or as a closed string tree-level cylinder diagram with one D-brane, represented by a closed string boundary state, emitting a closed string, propagating for a certain amount of time and finally absorbed by the other D-brane, also represented by a closed string boundary state. When the two D-branes are at rest, the net interaction vanishes by making use of the usual "abstruse identity" [3] and this goes by the name of "no-force" condition, which usually indicates that the underlying system preserves certain number of spacetime supersymmetries.

In addition to the simple strings or simple D-branes, i.e., extended objects charged under only one NS-NS potential or one R-R potential, there also exist their supersymmetry preserving bound states such as (F, D_p) [4–11] and (D_{p-2}, D_p) [12–14], i.e., extended objects charged under more than one potential. It would be interesting to know how to compute the forces between two such bound states separated by a distance. Since each of the bound states involves at least two kinds of branes, the force structure is richer and more interesting to explore. Our focus here will be on the above mentioned two types of the so-called non-threshold BPS bound states, namely (F, D_p) and (D_{p-2}, D_p), with even p in IIA and odd p in IIB, respectively.

The non-threshold BPS bound state (F, D_p), charged under both NS-NS 2-form potential and R-R $(p+1)$-form potential, is formed from the fundamental strings and D_p branes by lowering the system energy through dissolving the strings in the D_p branes, turning the strings into a worldvolume electric flux F_{0a} with the flux pointing along the direction of the original strings. The similar picture applies to the non-threshold BPS (D_{p-2}, D_p) bound state charged under both R-R $(p-1)$-form potential and R-R $(p+1)$-form potential, where the initial D_{p-2} branes dissolve in D_p branes, giving rise to a worldvolume magnetic flux F_{ab} with the spatial directions a and b pointing along the codimension-2 directions of the original D_{p-2} branes inside the D_p branes. Dirac charge quantization implies that the two potentials for either bound state are characterized by their corresponding quantized charges, therefore each bound state is characterized by a pair of integers (m, n). When the pair of integers is co-prime, the system is stable (otherwise it is marginally unstable) [15].

In a previous paper [1], the present authors along with the other two used the description of a boundary state with a quantized world-volume flux given in [11, 14, 18] for the bound state and computed the tree-level cylinder diagram interaction amplitude between two (F, D_p) or between two (D_{p-2}, D_p) bound states when the two bound states are placed parallel at a separation in a sense that the D_p branes in one bound state are along the same directions as those in the other bound state and so are the two fluxes. In the present paper, we will extend the computations to exhaust the remaining cases for which the two sets of D_p branes are still parallel at a separation but the two fluxes point differently. Concretely we will consider: 1) the two bound states are both (F, D_p) but with their respective non-vanishing quantized electric fluxes F_{0a} and F_{0b} pointing in a different direction, i.e., with $a \neq b$; 2) the two bound states are both (D_{p-2}, D_p) but with the respective non-vanishing quantized magnetic fluxes F_{ab} $(a < b)$ and F_{cd} $(c < d)$ sharing at most one common index, i.e., either $a = c$ but $b \neq d$ or $a = d$ or $b = d$ but $a \neq c$ or $a \neq c$ and $b \neq d$; and 3) one bound state is (F, D_p) and the other (D_{p-2}, D_p) with the electric flux F_{0a} pointing along either or neither of the two spatial indices of the magnetic flux F_{cd}, i.e., either $a = c$ or $a = d$ or $a \neq c, d$. When the two fluxes share one common index (either temporal or spatial) in each of the above three cases, we find that all the amplitudes have the similar structure to the one when the two fluxes share both of their two indices as given in [1], therefore with many features in common. However, when the two fluxes share no common index, the structure is different and more general, including the aforementioned one as a special case, therefore having more rich and interesting features.

Given each bound state characterized by a pair of integers (m_i, n_i) with $i = 1, 2$, we also find that the non-degenerate (i.e., $m_i n_i \neq 0$) force is in general attractive when the two fluxes are both magnetic or when one flux is magnetic and the other electric with the two sharing one common index and with the magnetic flux dominating over the electric flux in effect. However, we are certain that this force is attractive only at large separation when the two fluxes are both electric or when one flux is electric and the other magnetic either with the two sharing no common index or with the two sharing one common index and with the electric flux dominating over the magnetic flux in effect. When the two fluxes share one common index, the interaction amplitude can vanish only if there are one electric flux and one magnetic flux present and the string coupling is completely determined by the two pairs of the quantized charges with each characterizing the corresponding bound state. When the two fluxes share no common index, the amplitude can vanish only if the two fluxes are both magnetic and have the same magnitude. In either case, the underlying system preserves only 1/4 of space-time supersymmetries.

We also study the analytic structures of amplitudes under consideration and for the case with both fluxes magnetic or with the magnetic flux dominating over the electric flux in effect when the two shares one common index, the amplitude is real and diverges when the brane separation is on the order of string scale, signalling the onset of tachyonic instability. For each of the remaining cases, i.e., with one electric flux or at least one dominant electric flux present, the amplitude has an imaginary part and this gives rise to a non-vanishing rate for open string pair production. In particular, when the two fluxes share no common index, the rate of pair production of open strings is greatly enhanced by the presence of this fixed magnetic flux even when the electric flux is weak. This rate can be very significant even before the onset of tachyonic instability from the real part of the amplitude when the brane separation is on the order of string scale. Both this rate enhancement and the onset of tachyonic instability are not seen in a similar context when the two fluxes share at least one common index.

This paper is organized as follows. In the following section, we will give a very brief recall of the boundary state with a given external flux, providing the representation for the non-threshold (F, D_p) or (D_{p-2}, D_p) bound state. In section 3, we calculate the interaction amplitudes at the closed string tree-level cylinder diagram for those cases as specified above using the closed string boundary state approach with each state characterized by an arbitrary pair of integers (m_i, n_i) (i= 1, 2), and study the underlying properties. We summarize the results in section 4.

2 The boundary state

We in this section review very briefly what we need about the boundary state of D-branes with a constant external field on the world-volume and set the conventions for this paper. A rather complete account of this is given in [11, 14, 16–18].

In the closed string operator formalism, the supersymmetric BPS D-branes of type II theories can be described by means of boundary states $|B\rangle$ [19, 20]. For such a description, we have two sectors, namely NS-NS and R-R sectors, respectively. Both in the NS-NS and in R-R sectors, there are two possible implementations for the boundary conditions of a D-brane which correspond to two boundary states $|B, \eta\rangle$ with $\eta = \pm$. However, only the following combinations

$$|B\rangle_{\mathrm{NS}} = \frac{1}{2}\left[|B, +\rangle_{\mathrm{NS}} - |B, -\rangle_{\mathrm{NS}}\right], \tag{2.1}$$

and

$$|B\rangle_{\mathrm{R}} = \frac{1}{2}\left[|B, +\rangle_{\mathrm{R}} + |B, -\rangle_{\mathrm{R}}\right] \tag{2.2}$$

are selected by the GSO projection in the NS-NS and in the R-R sectors, respectively. The boundary state $|B, \eta\rangle$ is the product of a matter part and a ghost part [16] as

$$|B, \eta\rangle = \frac{c_p}{2}|B_{\mathrm{mat}}, \eta\rangle|B_{\mathrm{g}}, \eta\rangle, \tag{2.3}$$

where

$$|B_{\mathrm{mat}}, \eta\rangle = |B_X\rangle|B_\psi, \eta\rangle, \qquad |B_{\mathrm{g}}, \eta\rangle = |B_{gh}\rangle|B_{sgh}, \eta\rangle. \tag{2.4}$$

The overall normalization c_p can be unambiguously fixed from the factorization of amplitudes of closed strings emitted from a disk [14, 21] and is given by

$$c_p = \sqrt{\pi}\left(2\pi\sqrt{\alpha'}\right)^{3-p}. \tag{2.5}$$

As discussed in [11], the operator structure of the boundary state does not change even with the presence of an external flux on the worldvolume and is always of the form

$$|B_X\rangle = \exp\left[-\sum_{n=1}^{\infty}\frac{1}{n}\alpha_{-n}\cdot S\cdot\tilde{\alpha}_{-n}\right]|B_X\rangle^{(0)}, \tag{2.6}$$

and

$$|B_\psi,\eta\rangle_{\rm NS} = -{\rm i}\,\exp\left[i\eta\sum_{m=1/2}^{\infty}\psi_{-m}\cdot S\cdot\tilde{\psi}_{-m}\right]|0\rangle \tag{2.7}$$

for the NS-NS sector and

$$|B_\psi,\eta\rangle_{\rm R} = -\exp\left[i\eta\sum_{m=1}^{\infty}\psi_{-m}\cdot S\cdot\tilde{\psi}_{-m}\right]|B,\eta\rangle_{\rm R}^{(0)} \tag{2.8}$$

for the R-R sector.[1] The matrix S and the zero-mode contributions $|B_X\rangle^{(0)}$ and $|B,\eta\rangle_{\rm R}^{(0)}$ encode all information about the overlap equations that the string coordinates have to satisfy, which in turn depend on the boundary conditions of the open strings ending on the D-brane. They can be determined respectively [11, 19] as

$$S = \left(\left[(\eta-\hat{F})(\eta+\hat{F})^{-1}\right]_{\alpha\beta}, -\delta_{ij}\right), \tag{2.9}$$

$$|B_X\rangle^{(0)} = \sqrt{-\det\left(\eta+\hat{F}\right)}\,\delta^{9-p}(q^i-y^i)\prod_{\mu=0}^{9}|k^\mu=0\rangle, \tag{2.10}$$

for the bosonic sector, and

$$|B_\psi,\eta\rangle_{\rm R}^{(0)} = \left(C\Gamma^0\Gamma^1\cdots\Gamma^p\frac{1+{\rm i}\eta\Gamma_{11}}{1+{\rm i}\eta}U\right)_{AB}|A\rangle|\tilde{B}\rangle \tag{2.11}$$

for the R sector. In the above, the Greek indices $\alpha,\beta,\ldots$ label the world-volume directions $0,1,\ldots,p$ along which the D$_p$ brane extends, while the Latin indices $i,j,\ldots$ label the directions transverse to the brane, i.e., $p+1,\ldots,9$. We also define $\hat{F}=2\pi\alpha' F$ with F the external worldvolume field. Also in the above, we have denoted by y^i the positions of the D-brane along the transverse directions, by C the charge conjugation matrix and by U the following matrix

$$U(\hat{F}) = \frac{1}{\sqrt{-\det(\eta+\hat{F})}};\exp\left(-\frac{1}{2}\hat{F}_{\alpha\beta}\Gamma^\alpha\Gamma^\beta\right); \tag{2.12}$$

[1] The phases chosen in (2.7) and (2.8) are just for the convenience when we compute the couplings to various bulk massless modes.

JHEP09(2009)093

where the symbol ; ; means that one has to expand the exponential and then to anti-symmetrize the indices of the Γ-matrices. $|A\rangle|\tilde{B}\rangle$ stands for the spinor vacuum of the R-R sector. We would like to point out that the η in the above means either sign $\pm$ or the flat signature matrix $(-1, +1, \ldots, +1)$ on the world-volume and should not be confused from the content.

Note that the ghost and super-ghost fields are not affected by the type of the boundary conditions imposed, therefore the corresponding part of the boundary state remains the same as the one without the presence of an external worldvolume field and their explicit expressions can be found in [16]. We would like to point out that the boundary state must be written in the $(-1, -1)$ super-ghost picture in the NS-NS sector, and in the asymmetric $(-1/2, -3/2)$ picture in the R-R sector in order to saturate the super-ghost number anomaly of the disk [16, 22].

3 The interaction amplitude calculations

We now proceed to calculate the cylinder-diagram amplitude between any two of the non-threshold BPS (F, D_p) and/or (D_{p-2}, D_p) bound states at a separation Y using the boundary state approach for those cases as specified in the Introduction. In addition, we will use the results to discuss certain properties of the underlying systems such as the analytic structure of the respective amplitudes and calculate the rate of pair production of open strings in the open string channel for those cases involving at least one electric-like flux.

The interaction vacuum amplitude can be calculated via

$$\Gamma = \langle B(m_1, n_1)|D|B(m_2, n_2)\rangle \tag{3.1}$$

where the bound state with a constant world-volume field in each sector has been given in section 2 and is characterized by a pair of integers (m_i, n_i) with $i = 1, 2$ and D is the closed string propagator defined as

$$D = \frac{\alpha'}{4\pi}\int_{|z|\le 1}\frac{d^2 z}{|z|^2} z^{L_0}\bar{z}^{\tilde{L}_0}. \tag{3.2}$$

Here L_0 and $\tilde{L}_0$ are the respective left and right mover total zero-mode Virasoro generators of matter fields, ghosts and superghosts. For example, $L_0 = L_0^X + L_0^\psi + L_0^{gh} + L_0^{sgh}$ where $L_0^X, L_0^\psi, L_0^{gh}$ and L_0^{sgh} represent contributions from matter fields X^μ, matter fields ψ^μ, ghosts b and c, and superghosts β and γ, respectively, and their explicit expressions can be found in any standard discussion of superstring theories, for example in [17], therefore will not be presented here even though we will need them in our following calculations. The above total vacuum amplitude has contributions from both NS-NS and R-R sectors, respectively, and can be written as $\Gamma = \Gamma_{\rm NS} + \Gamma_{\rm R}$. In calculating either $\Gamma_{\rm NS}$ or $\Gamma_{\rm R}$, we need to keep in mind that the boundary state used should be the GSO projected one as given in eq. (2.1) or eq. (2.2). For this purpose, we need to calculate first the following amplitude

$$\Gamma(\eta', \eta) = \langle B^1, \eta'|D|B^2, \eta\rangle \tag{3.3}$$

in each sector with $\eta'\eta = \pm$ and $B^i = B(m_i, n_i)$. In doing the calculations, we can set $\tilde{L}_0 = L_0$ in the above propagator due to the fact that $\tilde{L}_0|B\rangle = L_0|B\rangle$, which can be used to simplify the calculations. Given the structure of the boundary state in eq. (2.3) and eq. (2.4), the amplitude $\Gamma(\eta', \eta)$ can be factorized as

$$\Gamma(\eta', \eta) = \frac{n_1 n_2 c_p^2}{4} \frac{\alpha'}{4\pi} \int_{|z|\leq 1} \frac{d^2 z}{|z|^2} A^X \, A^{bc} \, A^{\psi}(\eta', \eta) \, A^{\beta\gamma}(\eta', \eta), \tag{3.4}$$

where we have replaced the c_p in the boundary state given in section 2 by nc_p with n an integer to count the multiplicity of the D_p branes in the bound state. In the above,

$$\begin{aligned} A^X &= \langle B_X^1 | |z|^{2L_0^X} | B_X^2 \rangle, & A^{\psi}(\eta', \eta) &= \langle B_{\psi}^1, \eta' | |z|^{2L_0^{\psi}} | B_{\psi}^2, \eta \rangle, \\ A^{bc} &= \langle B_{gh}^1 | |z|^{2L_0^{gh}} | B_{gh}^2 \rangle, & A^{\beta\gamma}(\eta', \eta) &= \langle B_{sgh}^1, \eta' | |z|^{2L_0^{sgh}} | B_{sgh}^2, \eta \rangle. \end{aligned} \tag{3.5}$$

In order to perform the calculations using the boundary states given in (2.6)–(2.8), (2.10) and (2.11), we need to specify the worldvolume gauge field and the S-matrix given in (2.9) for both (F, D_p) and (D_{p-2}, D_p) bound states, respectively. Let us denote the boundary states $\langle B^1, \eta'|$ and $|B^2, \eta\rangle$ in evaluating the amplitude in (3.3) as BS1 and BS2, respectively. Without loss of generality, we can always choose the external flux $\hat{F}_1$ associated with BS1 the following way for simplicity. When this boundary state is the type of (F, D_p), we choose $\hat{F}_1$ as

$$\hat{F}_1 = \begin{pmatrix} 0 & -f_1 & & & & \\ f_1 & 0 & & & & \\ & & \cdot & & & \\ & & & \cdot & & \\ & & & & \cdot & \\ & & & & & 0 \end{pmatrix}_{(p+1)\times(p+1)} . \tag{3.6}$$

The corresponding longitudinal part of the S matrix as given in (2.9) is now

$$S_{1\alpha\beta} = \begin{pmatrix} -\frac{1+f_1^2}{1-f_1^2} & \frac{2f_1}{1-f_1^2} & & & & & \\ -\frac{2f_1}{1-f_2^2} & \frac{1+f_1^2}{1-f_1^2} & & & & & \\ & & 1 & & & & \\ & & & \cdot & & & \\ & & & & \cdot & & \\ & & & & & \cdot & \\ & & & & & & 1 \end{pmatrix}_{(p+1)\times(p+1)} . \tag{3.7}$$

While for the boundary state being (D_{p-2}, D_p), we choose the $\hat{F}_1$ as

$$\hat{F}_1 = \begin{pmatrix} 0 & & & & & \\ & \cdot & & & & \\ & & \cdot & & & \\ & & & \cdot & & \\ & & & & 0 & -f_1 \\ & & & & f_1 & 0 \end{pmatrix}_{(p+1)\times(p+1)} , \tag{3.8}$$

with now the quantized $f_1 = -m_1/n_1$. We then have the longitudinal part of the S matrix as

$$S_{1\alpha\beta} = \begin{pmatrix} -1 & & & & & & \\ & 1 & & & & & \\ & & \cdot & & & & \\ & & & \cdot & & & \\ & & & & \cdot & & \\ & & & & & \frac{1-f_1^2}{1+f_1^2} & \frac{2f_1}{1+f_1^2} \\ & & & & & -\frac{2f_1}{1+f_1^2} & \frac{1-f_1^2}{1+f_1^2} \end{pmatrix}_{(p+1)\times(p+1)} . \tag{3.9}$$

With the above choice for $\hat{F}_1$, the external worldvolume flux $\hat{F}_2$ for BPS2 shall be the following for those cases considered in this paper. When this boundary state is the type of (F, D$_p$), the only non-vanishing components are $(\hat{F}_2)_{0a} = -(\hat{F}_2)_{a0} = -f_2$ with the given spatial worldvolume index $a \neq 1$ when BS1 is also the same type but without such a restriction on this index when BPS1 is the type of (D$_{p-2}$, D$_p$). While for this boundary state being the type of (D$_{p-2}$, D$_p$), the only non-vanishing components are $(\hat{F}_2)_{bc} = -(\hat{F}_2)_{cb} = -f_2$ for given worldvolume spatial indices c and b ($c > b$) and with the only restriction $b \neq p-1$ when BS1 is of the same type but without any restriction when BS1 is the type of (F, D$_p$). Here each flux f_i with $i = 1, 2$ is quantized with a pair of integers (m_i, n_i) as discussed in [1], and is given for the case of electric flux as

$$f_i = -\frac{m_i}{\triangle^{1/2}_{e(m_i,n_i)}}, \tag{3.10}$$

where

$$\triangle_{e(m_i,n_i)} \equiv m_i^2 + \frac{n_i^2}{g_s^2} \tag{3.11}$$

with (m_i, n_i) a pair of integers, g_s the string coupling constant and the subscript 'e' representing the flux being electric, while for the case of magnetic flux

$$f_i = -\frac{m_i}{n_i}, \tag{3.12}$$

and for latter purpose we also define

$$\triangle_{m(m_i,n_i)} = m_i^2 + n_i^2, \tag{3.13}$$

with the subscript 'm' representing the flux being a magnetic one.

With the above preparations, we are now ready to perform rather straightforward calculations for the various matrix elements specified in (3.5) in either NS-NS or R-R sector for those cases under consideration, using (2.6)–(2.8), (2.10) and (2.11) for the boundary states with $\hat{F}$ and the matrix S as just described as well as the full expression of S as given in (2.9). The calculations[2] can be clarified as two types according to whether

[2]For $p = 3$, the interaction with only one boundary state carrying a particular form of flux was considered for a completely different purpose in [23]. An implicit expression for the interaction in general was given in [24] but the discussion on the R-R sector there is confusing and we don't agree in this part, in particular on the regularization procedure of zero-modes.

JHEP09(2009)093

the two worldvolume fluxes $(\hat{F}_1)_{\alpha\beta}$ and $(\hat{F}_2)_{\gamma\delta}$ discussed above have one common index (either temporal or spatial) or have no common index at all for which we will discuss each separately next.

3.1 Two fluxes with one common index

The common index for the two fluxes just mentioned can be along either a temporal or a spatial direction and for the present case we need the worldvolume spatial dimensions $p \geq 2$. For these cases, the corresponding amplitude has the same structure as the one obtained in [1] when the two fluxes share the same indices. For simplicity, let us denote the electric flux as 'e' and magnetic flux as 'm' and so all possibilities in the present case can be denoted as[3] (e, e), (e, m), (m, e) and (m, m) which represent that BS1 and BS2 are both of the type (F, D$_p$), BS1 the type of (F, D$_p$) while BS2 the type of (D$_{p-2}$, D$_p$), BS1 the type of (D$_{p-2}$, D$_p$) and BS2 the type of (F, D$_p$), and BS1 and BS2 both of the type (D$_{p-2}$, D$_p$), respectively. We have now the various matrix elements specified in (3.5) as

$$\begin{aligned} A^X &= C_F\, V_{p+1}\, e^{-\frac{Y^2}{2\pi\alpha' t}} \left(2\pi^2\alpha'\, t\right)^{-\frac{9-p}{2}} \prod_{n=1}^{\infty} \frac{1}{(1-\lambda|z|^{2n})(1-\lambda^{-1}|z|^{2n})(1-|z|^{2n})^8}, \\ A^{bc} &= |z|^{-2} \prod_{n=1}^{\infty} (1-|z|^{2n})^2, \end{aligned} \tag{3.14}$$

for both NS-NS and R-R sectors,

$$\begin{aligned} A^{\beta\gamma}_{\rm NS}(\eta',\eta) &= |z| \prod_{n=1}^{\infty} \frac{1}{(1+\eta'\eta\,|z|^{2n-1})^2}, \\ A^{\psi}_{\rm NS} &= \prod_{n=1}^{\infty} (1+\eta'\eta\,\lambda|z|^{2n-1})(1+\eta'\eta\,\lambda^{-1}\,|z|^{2n-1})(1+\eta'\eta\,|z|^{2n-1})^8, \end{aligned} \tag{3.15}$$

for NS-NS sector, and

$$A^{\beta\gamma}_{\rm R}(\eta',\eta) A^{\psi}_{\rm R}(\eta',\eta) = -2^4\,|z|^2\, D_F\, \delta_{\eta'\eta,+} \prod_{n=1}^{\infty} (1+\lambda\,|z|^{2n})(1+\lambda^{-1}\,|z|^{2n})(1+|z|^{2n})^6, \tag{3.16}$$

for the R-R sector. Note that we have $|z| = e^{-\pi t}$ above, the matrix elements for ghosts and superghosts are independent of the external fluxes as expected, and in (3.16) we have followed the prescription given in [16, 17] not to separate the contributions from matter fields ψ^μ and superghosts in the R-R sector in order to avoid the complication due to the respective zero modes. Also in the above, we have

$$D_F^{-1} = C_F = \begin{cases} \sqrt{(1-f_1^2)(1-f_2^2)} & \text{for } (e,e), \\ \sqrt{(1-f_1^2)(1+f_2^2)} & \text{for } (e,m), \\ \sqrt{(1+f_1^2)(1+f_2^2)} & \text{for } (m,m), \end{cases} \tag{3.17}$$

[3]Note that the (m, e) case can be obtained from the (e, m) case by sending $f_1 \to i f_1, f_2 \to i f_2$ in what follows. So for simplicity, we will not list this case explicitly. Note also that the (m, m) case can be similarly obtained from (e, e) case with the same replacements. However, we would like to discuss these two later cases explicitly since the force nature and other properties of these two cases are rather different.

JHEP09(2009)093

and

$$\lambda + \lambda^{-1} = 2(2D_F^2 - 1) = \begin{cases} 2\frac{1+f_1^2+f_2^2-f_1^2f_2^2}{(1-f_1^2)(1-f_2^2)} & \text{for } (e,e), \\ 2\frac{1+f_1^2-f_2^2+f_1^2f_2^2}{(1-f_1^2)(1+f_2^2)} & \text{for } (e,m), \\ 2\frac{1-f_1^2-f_2^2-f_1^2f_2^2}{(1+f_1^2)(1+f_2^2)} & \text{for } (m,m). \end{cases} \tag{3.18}$$

Note that in both equations above, the other cases list above can be obtained, for example, from the (e, e) case simply by sending f_i to its imaginary value if the corresponding flux is a magnetic one. For (e, e), $D_F > 1$ and for (m, m), $D_F < 1$. For (e, m) or (m, e), when $D_F > 1$ we say that the electric flux is dominant in effect while $D_F < 1$ we say that the magnetic flux is dominant in effect.

As discussed in [1], in calculating A^X and $A^\psi(\eta', \eta)$ as given explicitly above, we have made use of an important property for the S matrix

$$S^T{}_\mu{}^\rho S_\rho{}^\nu = \delta_\mu{}^\nu, \tag{3.19}$$

with T denoting the transpose. This property enables us to perform unitary transformations of the respective operators in the boundary states (2.6)–(2.8) such that the S matrix appearing, for example, in BS1 completely disappears, while leaving BS2 with a new S matrix as $S = S_2 S_1^T$, in the course of evaluating the respective A^X or A^ψ. This new S matrix shares the same property (3.19) as the original S_1 and S_2 do but its determinant is always equal to one. Therefore this S matrix under consideration can always be diagonalized to give two eigenvalues λ and λ^{-1} with their sum as given in (3.18) above and the other eight eigenvalues all equal to one. This is the basis for the structure appearing in the contributions due to the respective oscillators to the A^X and $A^\psi(\eta', \eta)$ as given in (3.14)–(3.16) above.

We can now have the vacuum amplitude in the NS-NS sector as

$$\begin{aligned} \Gamma_{\rm NS} &= {}_{\rm NS}\langle B^1|D|B^2\rangle_{\rm NS} \\ &= \frac{n_1 n_2 V_{p+1} C_F}{2(8\pi^2\alpha')^{\frac{1+p}{2}}} \int_0^\infty \frac{dt}{t}\, e^{-\frac{Y^2}{2\pi\alpha' t}}\, t^{-\frac{7-p}{2}} \\ &\quad \times |z|^{-1} \left[\prod_{n=1}^\infty \frac{(1+\lambda\,|z|^{2n-1})(1+\lambda^{-1}\,|z|^{2n-1})(1+|z|^{2n-1})^6}{(1-\lambda\,|z|^{2n})(1-\lambda^{-1}\,|z|^{2n})(1-|z|^{2n})^6} \right. \\ &\quad \left. - \prod_{n=1}^\infty \frac{(1-\lambda\,|z|^{2n-1})(1-\lambda^{-1}\,|z|^{2n-1})(1-|z|^{2n-1})^6}{(1-\lambda\,|z|^{2n})(1-\lambda^{-1}\,|z|^{2n})(1-|z|^{2n})^6} \right], \end{aligned} \tag{3.20}$$

where we have used the GSO projected boundary state in (2.1) for $|B^i\rangle_{\rm NS}$ (i = 1, 2) with B^i as defined previously and have made use of the matrix elements in (3.14) and (3.15) and the amplitude in (3.4). Also we have used in the above

$$\int_{|z|\le 1} \frac{d^2 z}{|z|^2} = 2\pi^2 \int_0^\infty dt, \tag{3.21}$$

with $|z| = e^{-\pi t}$. The corresponding vacuum amplitude in the R-R sector is now

$$\Gamma_R = {}_{\rm R}\langle B^1|D|B^2\rangle_{\rm R}$$

$$= -\frac{8\, n_1 n_2\, V_{p+1}}{(8\pi^2\alpha')^{\frac{1+p}{2}}} \int_0^\infty \frac{dt}{t}\, e^{-\frac{Y^2}{2\pi\alpha' t}}\, t^{-\frac{7-p}{2}}$$
$$\times \prod_{n=1}^{\infty} \frac{(1+\lambda\,|z|^{2n})(1+\lambda^{-1}\,|z|^{2n})(1+|z|^{2n})^6}{(1-\lambda\,|z|^{2n})(1-\lambda^{-1}\,|z|^{2n})(1-|z|^{2n})^6}, \tag{3.22}$$

where we have used the GSO projected boundary state in (2.2) for $|B^i\rangle_{\rm R}$ (i = 1, 2) again with B^i as defined previously and made use of the matrix elements in (3.14) and (3.16) and the amplitude in (3.4) as well as the equation (3.21). For both of (3.20) and (3.22), we have also made use of

$$\frac{c_p^2}{32\pi(2\pi^2\alpha')^{\frac{7-p}{2}}} = \frac{1}{(8\pi^2\alpha')^{\frac{p+1}{2}}} \times \frac{1}{2}, \tag{3.23}$$

where we have used the explicit expression (2.5) for c_p. We also always assume both n_1 and n_2 are positive integers and the p-branes in the non-threshold bound states are both D_p branes (or both anti D_p branes). In the case when the p-branes in either of the non-threshold bound states (but not both) are anti D_p branes, the corresponding $\Gamma_{\rm R}$ will switch sign from the one above but the $\Gamma_{\rm NS}$ will remain the same. In what follows, we will focus on that the p-branes in both non-threshold bound states are D_p-branes, i.e., (3.22) is valid. The case when the p-branes in either of the bound states are anti D_p-branes can be similarly analyzed.

The total vacuum amplitude is now

$$\begin{aligned}
\Gamma &= \Gamma_{\rm NS} + \Gamma_{\rm R}\\
&= \frac{n_1 n_2\, V_{p+1}\, C_F}{2(8\pi^2\alpha')^{\frac{1+p}{2}}} \int_0^\infty \frac{dt}{t}\, e^{-\frac{Y^2}{2\pi\alpha' t}}\, t^{-\frac{7-p}{2}}\\
&\quad\times \Bigg\{ |z|^{-1} \Bigg[\prod_{n=1}^{\infty} \frac{(1+\lambda\,|z|^{2n-1})(1+\lambda^{-1}\,|z|^{2n-1})(1+|z|^{2n-1})^6}{(1-\lambda\,|z|^{2n})(1-\lambda^{-1}\,|z|^{2n})(1-|z|^{2n})^6}\\
&\qquad\qquad - \prod_{n=1}^{\infty} \frac{(1-\lambda\,|z|^{2n-1})(1-\lambda^{-1}\,|z|^{2n-1})(1-|z|^{2n-1})^6}{(1-\lambda\,|z|^{2n})(1-\lambda^{-1}\,|z|^{2n})(1-|z|^{2n})^6} \Bigg]\\
&\qquad -2^4\, D_F \prod_{n=1}^{\infty} \frac{(1+\lambda\,|z|^{2n})(1+\lambda^{-1}\,|z|^{2n})(1+|z|^{2n})^6}{(1-\lambda\,|z|^{2n})(1-\lambda^{-1}\,|z|^{2n})(1-|z|^{2n})^6} \Bigg\},
\end{aligned} \tag{3.24}$$

which looks in form exactly the same as the one for the case when BS1 and BS2 are both of the same type, i.e., either (F, D_p) or (D_{p-2}, D_p), and the corresponding two worldvolume fluxes are along the same directions, as calculated in [1]. This is part of the basic result of this paper. This amplitude can also be expressed nicely in terms of θ-functions and the Dedekind η-function with their standard definitions as given, for example, in [27]. We then have

$$\begin{aligned}
\Gamma &= \frac{n_1 n_2\, V_{p+1}\, C_F\, \sin\pi\nu}{(8\pi^2\alpha')^{\frac{1+p}{2}}} \int_0^\infty \frac{dt}{t}\, e^{-\frac{Y^2}{2\pi\alpha' t}}\, t^{-\frac{7-p}{2}}\\
&\quad\times \frac{1}{\eta^9(it)} \left[\frac{\theta_3(\nu|it)\,\theta_3^3(0|it)}{\theta_1(\nu|it)} - \frac{\theta_4(\nu|it)\theta_4^3(0|it)}{\theta_1(\nu|it)} - \frac{\theta_2(\nu|it)\theta_2^3(0|it)}{\theta_1(\nu|it)} \right],
\end{aligned} \tag{3.25}$$

where we have defined $\lambda = e^{2\pi i\nu}$ and used the fact $\cos\pi\nu = D_F = C_F^{-1}$ which can be obtained from $\lambda + \lambda^{-1} = 2(2D_F^2 - 1)$ as given in (3.18) with C_F and D_F given in (3.17) . We also have

$$C_F \sin\pi\nu = \begin{cases} i\sqrt{f_1^2 + f_2^2 - f_1^2 f_2^2} & \text{for } (e,e), \\ \sqrt{-f_1^2 + f_2^2(1-f_1^2)} & \text{for } (e,m), \\ \sqrt{f_1^2 + f_2^2 + f_1^2 f_2^2} & \text{for } (m,m). \end{cases} \tag{3.26}$$

Note that for an electric flux $0 < |f_i| < 1$ while for a magnetic flux $0 < |f_i| < \infty$ and so $\nu = i\nu_0$ with $0 < \nu_0 < \infty$ for case (e, e), $\nu = \nu_0$ with $0 < \nu_0 < 1/2$ for case (m, m) but for (e, m), ν can be either real or imaginary depending on whether the magnetic flux or the electric flux dominates. For (e, m), when

$$|f_2| < \frac{|f_1|}{\sqrt{1-f_1^2}}, \tag{3.27}$$

the corresponding ν is imaginary, otherwise it will be real. Also only for this case (as well for (m, e) case), the corresponding amplitude can vanish with non-vanishing fluxes, which signals the preservation of certain number of corresponding spacetime supersymmetries. This actually occurs at (now $\nu = 0$)

$$f_2 = \pm\frac{f_1}{\sqrt{1-f_1^2}}, \tag{3.28}$$

which gives rise to a quantized string coupling as

$$g_s = \frac{n_1}{n_2}\frac{|m_2|}{|m_1|}. \tag{3.29}$$

To validate our computations, we need to have $g_s < 1$ which puts also constraint on the respective two pairs of integers above. As will be shown in the appendix, eq. (3.28) is precisely the condition for the underlying system to preserve also 1/4 of spacetime supersymmetries.

Our above amplitude can be greatly simplified if we make use of the following identity as discussed in [1] for θ-functions

$$2\,\theta_1^4(\nu|\tau) = \theta_3(2\nu|\tau)\,\theta_3^3(0|\tau) - \theta_4(2\nu|\tau)\,\theta_4^3(0|\tau) - \theta_2(2\nu|\tau)\,\theta_2^3(0|\tau), \tag{3.30}$$

and it is given by

$$\begin{aligned} \Gamma &= \frac{2\,n_1 n_2\,V_{p+1}\,C_F\,\sin\pi\nu}{(8\pi^2\alpha')^{\frac{1+p}{2}}}\int_0^\infty \frac{dt}{t}\,e^{-\frac{Y^2}{2\pi\alpha' t}}\,t^{-\frac{7-p}{2}}\,\frac{1}{\eta^9(it)}\,\frac{\theta_1^4(\frac{\nu}{2}|it)}{\theta_1(\nu|it)}, \\ &= \frac{2^4\,n_1 n_2\,V_{p+1} C_F\,\sin^4\frac{\pi\nu}{2}}{(8\pi^2\alpha')^{\frac{1+p}{2}}}\int_0^\infty \frac{dt}{t}\,e^{-\frac{Y^2}{2\pi\alpha' t}}\,t^{-\frac{7-p}{2}} \\ &\qquad\times\prod_{n=1}^{\infty}\frac{\left(1-e^{i\pi\nu}|z|^{2n}\right)^4\left(1-e^{-i\pi\nu}|z|^{2n}\right)^4}{\left(1-|z|^{2n}\right)^6\left(1-e^{2i\pi\nu}|z|^{2n}\right)\left(1-e^{-2i\pi\nu}|z|^{2n}\right)}, \end{aligned} \tag{3.31}$$

where in the second equality we have made use of explicit expressions for the Dedekind η-function and the theta-function θ_1 and in the above

$$\sin^4\frac{\pi\nu}{2} = \frac{1}{4}\,(\cos\pi\nu - 1)^2 = \frac{1}{4}(D_F - 1)^2. \tag{3.32}$$

We now consider the large Y limit of the amplitude (3.31) for $p \leq 6$. This amounts to accounting for the massless-mode contribution of closed string. Due to the exponential suppression of large Y, we need only to keep the leading-order contributions of the following in the integrand for large t,

$$\theta_1(\nu|it) \to 2e^{-\frac{\pi t}{4}}\sin\pi\nu, \qquad \theta_1(\frac{\nu}{2}|it) \to 2e^{-\frac{\pi t}{4}}\sin\frac{\pi\nu}{2}, \qquad \eta(it) \to e^{-\frac{\pi t}{12}}, \tag{3.33}$$

since now $|z| = e^{-\pi t} \to 0$. So

$$\begin{aligned}
\Gamma &\to \frac{2\,n_1 n_2\,V_{p+1}\,C_F\,\sin\pi\nu}{(8\pi^2\alpha')^{\frac{1+p}{2}}}\int_0^\infty\frac{dt}{t}\,e^{-\frac{Y^2}{2\pi\alpha' t}}\,t^{-\frac{7-p}{2}}\,\frac{1}{e^{-\frac{3\pi t}{4}}}\,\frac{2^4\,e^{-\pi t}\,\sin^4\frac{\pi\nu}{2}}{2\,e^{-\frac{\pi t}{4}}\,\sin\pi\nu},\\
&= \frac{2^4\,n_1 n_2\,V_{p+1}\,C_F\,\sin^4\frac{\pi\nu}{2}}{(8\pi^2\alpha')^{\frac{1+p}{2}}}\int_0^\infty\frac{dt}{t}\,e^{-\frac{Y^2}{2\pi\alpha' t}}\,t^{-\frac{7-p}{2}},\\
&= \frac{2^4\,n_1 n_2\,V_{p+1}\,C_F\,\sin^4\frac{\pi\nu}{2}}{(8\pi^2\alpha')^{\frac{1+p}{2}}}\left(\frac{2\pi\alpha'}{Y^2}\right)^{\frac{7-p}{2}}\Gamma\left(\frac{7-p}{2}\right),\\
&= \frac{C(m_1,n_1;m_2,n_2)}{Y^{7-p}}
\end{aligned} \tag{3.34}$$

where

$$C(m_1,n_1;m_2,n_2) = \frac{c_p^2\,V_{p+1}U(m_1,n_1;m_2,n_2)}{(7-p)\,\Omega_{8-p}}. \tag{3.35}$$

In the above, $(7-p)\Omega_{8-p} = 4\pi\pi^{(7-p)/2}/\Gamma((7-p)/2)$ with Ω_q the volume of unit q-sphere and

$$\begin{aligned}
U(m_1,n_1;m_2,n_2) &\equiv 4\,n_1 n_2 C_F\sin^4\frac{\pi\nu}{2},\\
&= \begin{cases}
\frac{\left(n_1 n_2 - g_s^2\Omega_{ee}\right)^2}{g_s^2\,\Omega_{ee}} & \text{for } (e,e),\\[2ex]
\frac{\left(g_s n_2\,\triangle^{1/2}_{e(m_1,n_1)} - n_1\triangle^{1/2}_{m(m_2,n_2)}\right)^2}{g_s\,\Omega_{em}} & \text{for } (e,m),\\[2ex]
\frac{(n_1 n_2 - \Omega_{mm})^2}{\Omega_{mm}} & \text{for } (m,m),
\end{cases}
\end{aligned} \tag{3.36}$$

where $\Omega_{ee} = \triangle^{1/2}_{e(m_1,n_1)}\triangle^{1/2}_{e(m_2,n_2)}$, $\Omega_{em} = \triangle^{1/2}_{e(m_1,n_1)}\triangle^{1/2}_{m(m_2,n_2)}$ and $\Omega_{mm} = \triangle^{1/2}_{m(m_1,n_1)}\triangle^{1/2}_{m(m_2,n_2)}$ with $\triangle_{e(m_i,n_i)}$ and $\triangle_{m(m_i,n_i)}$ defined in (3.11) and (3.13), respectively. For a non-vanishing flux, either electric or magnetic, i.e, $m_i \neq 0$ $(i = 1,2)$, the above $U(m_1,n_1;m_2,n_2)$ can vanish only for the case (e, m) (or (m, e)) and if this occurs, the corresponding amplitude as well as its large separation limit vanishes. The condition for this to occur is exactly the same as the one given in (3.29).

We will have $U(m_1, n_1; m_2, n_2) > 0$ in all cases considered above if the string coupling constant is not quantized as given in (3.29) for the case of (e, m). Note that each numerator in the infinite product in the integrand in the second equality of (3.31) can be re-expressed as

$$\left(1 - e^{i\pi\nu}|z|^{2n}\right)^4 \left(1 - e^{-i\pi\nu}|z|^{2n}\right)^4 = \left(1 - 2\cos\pi\nu\, |z|^{2n} + |z|^{4n}\right)^4 > 0, \tag{3.37}$$

so the sign of the interaction amplitude will depend on that of the factor in each denominator in the infinite product in the integrand

$$\left(1 - e^{2i\pi\nu}|z|^{2n}\right)\left(1 - e^{-2i\pi\nu}|z|^{2n}\right) = \left(1 - 2\cos 2\pi\nu\, |z|^{2n} + |z|^{4n}\right), \tag{3.38}$$

which is always positive for the case of (m, m) and the case of (e, m) when (3.27) is not satisfied, respectively. In other words, for the later case, the magnetic flux plays at least the equally important role as the electric flux and now ν is real. Then the corresponding interaction amplitude in each of the above two cases is greater than zero and is solely determined by the positiveness of $U(m_1, n_1; m_2, n_2)$. In this aspect it shares the same feature as its long distance interaction, reflecting the attractive nature of the interaction. While this factor is still positive for large t but it can be negative for small t for the case of (e, e) and the case of (e, m) when (3.27) is satisfied, respectively. In other words, for the (e, m) case, the electric flux now plays a dominant role and ν is now imaginary. For either of the present two cases, while the long distance interaction amplitude is again greater than zero (implying also an attractive interaction) and is also solely determined by the positiveness of the corresponding $U(m_1, n_1; m_2, n_2)$, the sign of the small separation amplitude (corresponding to small t contribution) is uncertain in the present representation of integration variable t since even with the factor in (3.38) less than zero, the sign of the product of infinite number of such factors in the integrand remains indefinite. So one expects some interesting physics to appear in this case for small t.

The small t contribution to the amplitude mainly concerns about the physics for small separation Y. The appropriate frame for describing the underlying physics as well as the analytic structure as a function of the separation in the short cylinder limit $t \to 0$ is in terms of an annulus, which can be achieved by the Jacobi transformation $t \to t' = 1/t$. This is also stressed in [29] that the lightest open string modes now contribute most and the open string description is most relevant. So in terms of the annulus variable t', noting

$$\begin{aligned}
\eta(\tau) &= \frac{1}{(-i\tau)^{1/2}}\eta\left(-\frac{1}{\tau}\right), \\
\theta_1(\nu|\tau) &= i\frac{e^{-i\pi\nu^2/\tau}}{(-i\tau)^{1/2}}\theta_1\left(\frac{\nu}{\tau}\Big| -\frac{1}{\tau}\right),
\end{aligned} \tag{3.39}$$

the amplitude in (3.31) can now be reexpressed as

$$\Gamma = -i\frac{U(m_1, n_1; m_2, n_2)\, V_{p+1}}{2(8\pi^2\alpha')^{\frac{1+p}{2}}}\frac{\sin\pi\nu}{\sin^4\frac{\pi\nu}{2}}\int_0^\infty \frac{dt'}{t'}\, e^{-\frac{Y^2 t'}{2\pi\alpha'}}\, t'^{\frac{1-p}{2}}\frac{1}{\eta^9(it')}\frac{\theta_1^4(\frac{-i\nu t'}{2}|it')}{\theta_1(-i\nu t'|it')},$$

$$= -i\frac{4\,U(m_1,n_1;m_2,n_2)\,V_{p+1}}{(8\pi^2\alpha')^{\frac{1+p}{2}}}\frac{\sin\pi\nu}{\sin^4\frac{\pi\nu}{2}}\int_0^\infty\frac{dt'}{t'}\,e^{-\frac{Y^2t'}{2\pi\alpha'}}\,t'^{\frac{1-p}{2}}\frac{\sin^4\left(\frac{-i\pi\nu t'}{2}\right)}{\sin\left(-i\pi\nu t'\right)}$$

$$\times\prod_{n=1}^{\infty}\frac{\left(1-e^{\pi\nu t'}|z|^{2n}\right)^4\left(1-e^{-\pi\nu t'}|z|^{2n}\right)^4}{(1-|z|^{2n})^6\,(1-e^{2\pi\nu t'}|z|^{2n})\,(1-e^{-2\pi\nu t'}|z|^{2n})},\tag{3.40}$$

where we have made use of the expression for $U(m_1,n_1;m_2,n_2)$ given in (3.36) and now $|z| = e^{-\pi t'}$. We follow [1, 26] to discuss the underlying analytic structure and the possible associated physics of the amplitude of (3.40). For the case when ν is real as mentioned above, we limit ourselves to the interesting non-BPS amplitude, i.e., $\nu = \nu_0$ with $0 < \nu_0 < 1/2$, and for this the above amplitude is purely real and has no singularities unless $Y \leq \pi\sqrt{2\nu\alpha'}$, i.e. on the order of string scale, for which the integrand is dominated by, in the short cylinder limit $t' \to \infty$,

$$\lim_{t'\to\infty}\frac{e^{-\frac{Y^2t'}{2\pi\alpha'}}\theta_1(-i\pi\nu t'/2|it')}{i\,\eta(it')\theta_1(-i\pi\nu t'|it')}\sim\lim_{t'\to\infty}\frac{e^{-\frac{Y^2t'}{2\pi\alpha'}}\sin^4(-i\pi\nu t'/2)}{i\,\sin(-i\pi\nu t')}\sim\lim_{t'\to\infty}e^{-\frac{t'}{2\pi\alpha'}(Y^2-2\pi^2\nu\alpha')}.\tag{3.41}$$

The contribution of the annulus to the vacuum amplitude (energy) should be real if the integrand in (3.40) have no simple poles on the positive t'-axis since the imaginary part of the amplitude is given by the sum of residues at the poles times π due to the integration contour passing to the right of all poles as dictated by the proper definition of the Feynman propagator [30]. In the present case, the amplitude appears purely real and there are no simple poles on the positive t'-axis, therefore giving zero imaginary amplitude, i.e., zero pair-production (absorptive) rate of open strings, which is consistent with the conclusion reached in [31] in quantum field theory context and also pointed out in a similar context in [1, 32]. When $Y \leq \pi\sqrt{2\nu_0\alpha'}$, i.e., on the order of string scale, the amplitude diverges as indicated in (3.44) and this happens in a similar fashion as in the case of a brane/antibrane system as discussed in [39, 40] but now caused by the presence of dominant magnetic flux or fluxes. The appearance of the divergent amplitude indicates the breakdown of the calculations, signalling the onset of tachyonic instability caused by the dominant magnetic flux or fluxes[4] and the relaxation of the system to form a new non-threshold bound state. However, the detail of this requires further dynamical understanding.

Let us move to the case when the electric flux or fluxes are dominant in a sense mentioned earlier. We have now $\nu = i\nu_0$ with $0 < \nu_0 < \infty$ ($\nu_0 = 0$ corresponds to BPS

[4]With $0 < \nu = \nu_0 < 1/2$, in addition to the evidence given in the text, that the open string tachyon mode appears to arise is also indicated from the leading term $e^{\pi\nu t'}$, which diverges in the short cylinder limit $t' \to \infty$, in the expansion of the θ-functions and η-function in (3.40) in the open string channel. Note also that in the case of (e, m), ν can be zero when the string coupling satisfies (3.29) and this divergent term, therefore the tachyon mode, then disappears and this is entirely consistent with the fact that the amplitude also vanishes, signalling the underlying system being BPS and preserving certain number of spacetime supersymmetries.

JHEP09(2009)093

case and is not considered here). The amplitude (3.40) is now

$$\Gamma = \frac{4\,U(m_1,n_1;m_2,n_2)\,V_{p+1}}{(8\pi^2\alpha')^{\frac{1+p}{2}}}\frac{\sinh\pi\nu_0}{\sinh^4\frac{\pi\nu_0}{2}}\int_0^\infty \frac{dt'}{t'}\,e^{-\frac{Y^2 t'}{2\pi\alpha'}}\,t'^{\frac{1-p}{2}}\frac{\sin^4\left(\frac{\pi\nu_0 t'}{2}\right)}{\sin(\pi\nu_0 t')}$$
$$\times\prod_{n=1}^{\infty}\frac{\left(1-e^{i\pi\nu_0 t'}|z|^{2n}\right)^4\left(1-e^{-i\pi\nu_0 t'}|z|^{2n}\right)^4}{(1-|z|^{2n})^6\,(1-e^{2i\pi\nu_0 t'}|z|^{2n})\,(1-e^{-2i\pi\nu_0 t'}|z|^{2n})}. \tag{3.42}$$

Exactly the same as the cases discussed in [1, 26], the above integrand has also an infinite number of simple poles on the positive real t'-axis at $t' = (2k+1)/\nu_0$ with $k = 0, 1, 2, \ldots$. This leads to an imaginary part of the amplitude, which is given as the sum over the residues of the poles as described in [30, 33]. Therefore the rate of pair production of open strings per unit worldvolume in the present context is

$$\begin{aligned}
\mathcal{W} &\equiv -\frac{2\mathrm{Im}\Gamma}{V_{p+1}},\\
&= \frac{8\,U(m_1,n_1;m_2,n_2)}{\nu_0(8\pi^2\alpha')^{\frac{1+p}{2}}}\frac{\sinh\pi\nu_0}{\sinh^4\frac{\pi\nu_0}{2}}\sum_{k=0}^{\infty}\left(\frac{\nu_0}{2k+1}\right)^{\frac{1+p}{2}}e^{-\frac{(2k+1)Y^2}{2\pi\nu_0\alpha'}}\prod_{n=1}^{\infty}\left(\frac{1+e^{-2n\pi(2k+1)/\nu_0}}{1-e^{-2n(2k+1)\pi/\nu_0}}\right)^8,\\
&= \frac{32 n_1 n_2\tanh\pi\nu_0}{\nu_0(8\pi^2\alpha')^{\frac{1+p}{2}}}\sum_{k=0}^{\infty}\left(\frac{\nu_0}{2k+1}\right)^{\frac{1+p}{2}}e^{-\frac{(2k+1)Y^2}{2\pi\nu_0\alpha'}}\prod_{n=1}^{\infty}\left(\frac{1+e^{-2n(2k+1)\pi/\nu_0}}{1-e^{-2n(2k+1)\pi/\nu_0}}\right)^8,
\end{aligned} \tag{3.43}$$

where we have used $U(m_1,n_1;m_2,n_2) = 4\,n_1 n_2 C_F\sinh^4\pi\nu_0/2$ in the present context and ν_0 can be determined from

$$\tanh\pi\nu_0 = \begin{cases}\dfrac{\sqrt{g_s^2 m_1^2 m_2^2 + n_1^2 m_2^2 + n_2^2 m_1^2}}{g_s\,\Omega_{ee}} & \text{for } (e,e),\\[2ex] \dfrac{\sqrt{m_1^2 n_2^2 g_s^2 - n_1^2 m_2^2}}{g_s n_2 \triangle^{1/2}_{e(m_1,n_1)}} & \text{for } (e,m),\end{cases} \tag{3.44}$$

where $\triangle_{e(m_i,n_i)}$ is defined in (3.11) with $i = 1,2$ and $\Omega_{ee} = \triangle^{1/2}_{e(m_1,n_1)}\triangle^{1/2}_{e(m_2,n_2)}$ as defined earlier. Also in the above, the condition (3.27) for the case of (e, m) needs to be satisfied. In the present context, it is $g_s > n_1|m_2|/(n_2|m_1|)$. Note that the above rate is suppressed by the brane separation and the integer k but increases with the vlue of ν_0 which is expected. Let us consider $\nu_0 \to 0$ and $\nu_0 \to \infty$ limits for each case considered above. The former limit corresponds to the near extremal limit which requires both electric fluxes to be small or equivalently $n_i \gg g_s m_i$ with $i = 1, 2$ for the case of (e, e) and $(g_s n_2|m_1| - n_1|m_2|) \to 0^+$ for the case of (e, m). In either of these two cases, $\nu_0 \to 0$ and $\tanh\pi\nu_0 \to \pi\nu_0$. The rate is now approximated well by the leading $k = 0$ term as

$$\mathcal{W} \approx 32 n_1 n_2\,\pi\left(\frac{\nu_0}{8\pi^2\alpha'}\right)^{\frac{1+p}{2}} e^{-\frac{Y^2}{2\pi\nu_0\alpha'}}, \tag{3.45}$$

vanishing small as expected. The $\nu_0 \to \infty$ limit requires that the electric flux or fluxes all reach their respective critical field limit in either case considered here. In addition, we need $g_s\,|m_1|/n_1 \gg |m_2|/n_2$ for the case of (e, m). Then each term in the summation of (3.43) diverges and so does the rate, signalling also an instability as mentioned in a similar context in [35].

3.2 Two fluxes without common index

We now discuss the cases when the two non-vanishing worldvolume constant fluxes $(\hat F_1)_{\alpha\beta}$ and $(\hat F_2)_{\gamma\delta}$ specified at the beginning of this section share no common index, i.e., $\alpha, \beta \neq \gamma, \delta$. So we have only three cases to consider: 1) (e, m), 2) (m, e) and 3) (m, m). For the former two cases, we need $p \geq 3$ for the spatial dimensions of Dp branes in the non-threshold bound states while for the later case, we need $p \geq 4$. If $(\hat F_1)_{\alpha\beta}$ is an electric flux as specified in (3.6), then we have the case 1) above with $(\hat F_2)_{\gamma\delta}$ a magnetic flux. We choose then its only two non-vanishing components $(\hat F_2)_{cd} = -(\hat F_2)_{dc} = -f_2$ at two given spatial indices c, d with the constraint $c < d$ and $c \neq 1$. If $(\hat F_1)_{\alpha\beta}$ is a magnetic flux as specified in (3.8), we can have either case 2) above with the only two non-vanishing electric flux components $(\hat F_2)_{0a} = -(\hat F_2)_{a0} = -f_2$ with $a \neq p-1, p$ or the case 3) with the only two non-vanishing magnetic components $(\hat F_2)_{cd} = -(\hat F_2)_{dc} = -f_2$ with now the spatial indices $c < d$ and $c, d \neq p-1, p$. We have then the various matrix elements specified in (3.5) as

$$\begin{aligned} A^X &= C_F\, V_{p+1}\, e^{-\frac{Y^2}{2\pi\alpha' t}} \left(2\pi^2\alpha'\, t\right)^{-\frac{9-p}{2}} \prod_{n=1}^{\infty} \frac{1}{(1-|z|^{2n})^6} \prod_{j=1}^{2} \frac{1}{(1-\lambda_j|z|^{2n})(1-\lambda_j^{-1}|z|^{2n})},\\ A^{bc} &= |z|^{-2} \prod_{n=1}^{\infty} (1-|z|^{2n})^2, \end{aligned} \tag{3.46}$$

for both NS-NS and R-R sectors,

$$\begin{aligned} A^{\beta\gamma}_{\rm NS}(\eta', \eta) &= |z| \prod_{n=1}^{\infty} \frac{1}{(1+\eta'\eta\,|z|^{2n-1})^2},\\ A^{\psi}_{\rm NS} &= \prod_{n=1}^{\infty} (1+\eta'\eta\,|z|^{2n-1})^6 \prod_{j=1}^{2} (1+\eta'\eta\,\lambda_j |z|^{2n-1})(1+\eta'\eta\,\lambda_j^{-1}\,|z|^{2n-1}), \end{aligned} \tag{3.47}$$

for NS-NS sector, and

$$A^{\beta\gamma}_{\rm R}(\eta', \eta) A^{\psi}_{\rm R}(\eta', \eta) = -2^4\,|z|^2\, D_F\, \delta_{\eta'\eta, +} \prod_{n=1}^{\infty} (1+|z|^{2n})^4 \prod_{j=1}^{2} (1+\lambda_j\,|z|^{2n})(1+\lambda_j^{-1}\,|z|^{2n}), \tag{3.48}$$

for the R-R sector. Note that we have $|z| = e^{-\pi t}$ above and again in (3.48) we follow the prescription given in [16, 17] not to separate the contributions from matter fields ψ^μ and superghosts in the R-R sector in order to avoid the complication due to the respective zero modes. Also in the above,[5] we have

$$D_F^{-1} = C_F = \begin{cases} \sqrt{(1-f_1^2)(1+f_2^2)} & \text{for } (e, m),\\ \sqrt{(1+f_1^2)(1+f_2^2)} & \text{for } (m, m), \end{cases} \tag{3.49}$$

[5] By the same token, the (m, e) case can be similarly discussed, therefore not repeated in what follows for simplicity.

and

$$\lambda_1 + \lambda_1^{-1} = 2(2D_{F_1}^2 - 1) = \begin{cases} 2\frac{1+f_1^2}{1-f_1^2} & \text{for } (e,m), \\ \\ 2\frac{1-f_1^2}{1+f_1^2} & \text{for } (m,m), \end{cases} \tag{3.50}$$

$$\lambda_2 + \lambda_2^{-1} = 2(2D_{F_2}^2 - 1) = \begin{cases} 2\frac{1-f_2^2}{1+f_2^2} & \text{for } (e,m), \\ \\ 2\frac{1-f_2^2}{1+f_2^2} & \text{for } (m,m), \end{cases} \tag{3.51}$$

where $D_F = D_{F_1} D_{F_2}$.

With the above matrix elements and by using (3.3) and (3.4), we can have the amplitude in the NS-NS sector using (3.1) with the GSO projected boundary state in the NS-NS sector defined in (2.1) as

$$\begin{aligned} \Gamma_{\rm NS} = & \frac{n_1 n_2 V_{p+1} C_F}{2(8\pi^2\alpha')^{\frac{1+p}{2}}} \int_0^\infty \frac{dt}{t} e^{-\frac{Y^2}{2\pi\alpha' t}} t^{-\frac{7-p}{2}} \\ & \times |z|^{-1} \left[\prod_{n=1}^\infty \frac{(1+|z|^{2n-1})^4}{(1-|z|^{2n})^4} \prod_{j=1}^2 \frac{(1+\lambda_j |z|^{2n-1})(1+\lambda_j^{-1}|z|^{2n-1})}{(1-\lambda_j |z|^{2n})(1-\lambda_j^{-1}|z|^{2n})} \right. \\ & \left. - \prod_{n=1}^\infty \frac{(1-|z|^{2n-1})^4}{(1-|z|^{2n})^4} \prod_{j=1}^2 \frac{(1-\lambda_j |z|^{2n-1})(1-\lambda_j^{-1}|z|^{2n-1})}{(1-\lambda_j |z|^{2n})(1-\lambda_j^{-1}|z|^{2n})} \right], \end{aligned} \tag{3.52}$$

and similarly we have the amplitude in the R-R sector using the GSO projected boundary state given in (2.2) as

$$\begin{aligned} \Gamma_R = & -\frac{8\, n_1 n_2 V_{p+1}}{(8\pi^2\alpha')^{\frac{1+p}{2}}} \int_0^\infty \frac{dt}{t} e^{-\frac{Y^2}{2\pi\alpha' t}} t^{-\frac{7-p}{2}} \\ & \times \prod_{n=1}^\infty \frac{(1+|z|^{2n})^4}{(1-|z|^{2n})^4} \prod_{j=1}^2 \frac{(1+\lambda_j |z|^{2n})(1+\lambda_j^{-1}|z|^{2n})}{(1-\lambda_j |z|^{2n})(1-\lambda_j^{-1}|z|^{2n})}. \end{aligned} \tag{3.53}$$

In obtaining the above amplitudes, we have made use of equations (3.21) and (3.23) and some cautions mentioned in the previous subsection below (3.23) also apply and will not be repeated here.

The total amplitude is then

$$\begin{aligned} \Gamma = & \Gamma_{\rm NS} + \Gamma_{\rm R} \\ = & \frac{n_1 n_2 V_{p+1} C_F}{2(8\pi^2\alpha')^{\frac{1+p}{2}}} \int_0^\infty \frac{dt}{t} e^{-\frac{Y^2}{2\pi\alpha' t}} t^{-\frac{7-p}{2}} \\ & \times \left\{ |z|^{-1} \left[\prod_{n=1}^\infty \frac{(1+|z|^{2n-1})^4}{(1-|z|^{2n})^4} \prod_{j=1}^2 \frac{(1+\lambda_j |z|^{2n-1})(1+\lambda_j^{-1}|z|^{2n-1})}{(1-\lambda_j |z|^{2n})(1-\lambda_j^{-1}|z|^{2n})} \right. \right. \end{aligned}$$

$$
-\prod_{n=1}^{\infty}\frac{(1-|z|^{2n-1})^4}{(1-|z|^{2n})^4}\prod_{j=1}^{2}\frac{(1-\lambda_j\,|z|^{2n-1})(1-\lambda_j^{-1}\,|z|^{2n-1})}{(1-\lambda_j\,|z|^{2n})(1-\lambda_j^{-1}\,|z|^{2n})}\Bigg]
$$

$$
-2^4 D_F\prod_{n=1}^{\infty}\frac{(1+|z|^{2n})^4}{(1-|z|^{2n})^4}\prod_{j=1}^{2}\frac{(1+\lambda_j\,|z|^{2n})(1+\lambda_j^{-1}\,|z|^{2n})}{(1-\lambda_j\,|z|^{2n})(1-\lambda_j^{-1}\,|z|^{2n})}\Bigg\}, \tag{3.54}
$$

where the structure looks a bit different from the one given in the previous subsection and in [1] for which the two fluxes share at least one common direction. As we will show, the previous structure for amplitudes is just a special case of the present one. The above amplitude can be re-expressed in a nice form in terms of various θ-functions and the Dedekind η-function with their standard definitions as mentioned in the previous subsection. We then have

$$
\begin{aligned}
\Gamma =& \frac{2\,n_1 n_2\,V_{p+1}\,\tan\pi\nu_1\,\tan\pi\nu_2}{(8\pi^2\alpha')^{\frac{1+p}{2}}}\int_0^\infty\frac{dt}{t}\,e^{-\frac{Y^2}{2\pi\alpha' t}}\,t^{-\frac{7-p}{2}}\\
&\times\frac{1}{\eta^6(it)}\left[\frac{\theta_3(\nu_1|it)\,\theta_3(\nu_2|it)\,\theta_3^2(0|it)}{\theta_1(\nu_1|it)\,\theta_1(\nu_2|it)}-\frac{\theta_4(\nu_1|it)\,\theta_4(\nu_2|it)\,\theta_4^2(0|it)}{\theta_1(\nu_1|it)\theta_1(\nu_2|it)}\right.\\
&\left.-\frac{\theta_2(\nu_1|it)\,\theta_2(\nu_2|it)\,\theta_2^2(0|it)}{\theta_1(\nu_1|it)\,\theta_1(\nu_2|it)}\right],
\end{aligned}\tag{3.55}
$$

where we have defined $\lambda_j = e^{2\pi i\nu_j}$ and used the fact $\cos\pi\nu_j = D_{F_j}$ which can be obtained from $\lambda_j+\lambda_j^{-1} = 2(2D_{F_j}^2-1)$ as given in (3.50) and (3.51) with $D_{F_j} = 1/\sqrt{1-f_j^2}$ for an electric flux and $D_{F_j} = 1/\sqrt{1+f_j^2}$ for a magnetic flux for $j = 1, 2$, respectively. We also have

$$
\tan\pi\nu_j = \begin{cases} i|f_j| & \text{for an electric flux,}\\[2ex] |f_j| & \text{for a magnetic flux,}\end{cases}\tag{3.56}
$$

where the subscript index $j = 1, 2$. Note that for an electric flux $0 < |f_j| < 1$ while for a magnetic flux $0 < |f_j| < \infty$ and so $\nu_j = i\nu_{j0}$ with $0 < \nu_{j0} < \infty$ for an electric flux and $\nu_j = \nu_{j0}$ with $0 < \nu_{j0} < 1/2$ for a magnetic flux. The above amplitude can be greatly simplified if the following identity for θ-functions is employed[6]

$$
\begin{aligned}
2\,\theta_1^2\left(\frac{\nu_1-\nu_2}{2}\middle|\tau\right)\theta_1^2\left(\frac{\nu_1+\nu_2}{2}\middle|\tau\right) =& \;\theta_3(\nu_1|\tau)\theta_3(\nu_2|\tau)\theta^2(0|\tau)-\theta_4(\nu_1|\tau)\theta_4(\nu_2|\tau)\theta_4^2(0|\tau)\\
&-\theta_2(\nu_1|\tau)\theta_2(\nu_2|\tau)\theta_2(0|\tau),
\end{aligned}\tag{3.57}
$$

and the amplitude becomes

$$
\Gamma = \frac{4\,n_1 n_2\,V_{p+1}\,\tan\pi\nu_1\,\tan\pi\nu_2}{(8\pi^2\alpha')^{\frac{1+p}{2}}}\int_0^\infty\frac{dt}{t}\,e^{-\frac{Y^2}{2\pi\alpha' t}}\,t^{-\frac{7-p}{2}}\,\frac{1}{\eta^6(it)}\,\frac{\theta_1^2\left(\frac{\nu_1-\nu_2}{2}\middle|\,it\right)\,\theta_1^2\left(\frac{\nu_1+\nu_2}{2}\middle|\,it\right)}{\theta_1(\nu_1|it)\theta_1(\nu_2|it)},
$$

[6]This identity can be obtained from the general one (iv) on page 468 given in [28]. The notations there for various θ-functions are $\theta_r(z) \equiv \theta_r(z|\tau)$ with $r = 1, 2, 3, 4$. In obtaining (3.57) in the text, we need to make choices for the variables as $y' = 0, z' = 0, w' = x + y, x' = -x + y$ and set $x = (\nu_1-\nu_2)/2, y = (\nu_1+\nu_2)/2$.

$$= \frac{2^4\, n_1 n_2\, V_{p+1}\, C_F \sin^2\frac{\pi(\nu_1-\nu_2)}{2}\, \sin^2\frac{\pi(\nu_1+\nu_2)}{2}}{(8\pi^2\alpha')^{\frac{1+p}{2}}} \int_0^\infty \frac{dt}{t}\, e^{-\frac{Y^2}{2\pi\alpha' t}}\, t^{-\frac{7-p}{2}}$$

$$\times \prod_{n=1}^{\infty} \frac{1}{(1-|z|^{2n})^4} \prod_{j=1}^{2} \frac{(1-e^{\pi i(\nu_1+(-)^j\nu_2)}|z|^{2n})^2(1-e^{-\pi i(\nu_1+(-)^j\nu_2)}|z|^{2n})^2}{(1-e^{2\pi i\nu_j}|z|^{2n})(1-e^{-2\pi i\nu_j}|z|^{2n})}, \quad (3.58)$$

where in the second equality we have used the explicit expression for $\theta_1(\nu|\tau)$ and $C_F^{-1} = D_F = D_{F_1} D_{F_2}$ with $D_{F_j} = \cos\pi\nu_j$. One can check easily with the known properties of $\theta_1(\nu|\tau)$ that both (3.58) and (3.55) will reduce their basic structures to their corresponding ones obtained in the previous subsection or in [1] where the two worldvolume fluxes share at least one common direction if we set either ν_1 or $\nu_2 \to 0$ in (3.58) and (3.55). Therefore the basic structure of either (3.58) or (3.55) is more general than their respective previous correspondence just mentioned. For later purpose, let us define the following quantity similar to (3.36) as

$$U(m_1,n_1;m_2,n_2) \equiv 4\, n_1 n_2 C_F \sin^2\frac{\pi(\nu_1-\nu_2)}{2}\sin^2\frac{\pi(\nu_1+\nu_2)}{2} = n_1 n_2 \frac{(D_{F_1}-D_{F_2})^2}{D_{F_1}D_{F_2}},$$

$$= \begin{cases} \frac{(n_1 n_2 - g_s \Omega_{em})^2}{g_s\, \Omega_{em}} & \text{for } (e,m), \\ \frac{\left(n_1 \triangle^{1/2}_{m(m_2,n_2)} - n_2 \triangle^{1/2}_{m(m_1,n_1)}\right)^2}{\Omega_{mm}} & \text{for } (m,m), \end{cases} \quad (3.59)$$

where $\Omega_{em} = \triangle^{1/2}_{e(m_1,n_1)}\triangle^{1/2}_{m(m_2,n_2)}$ and $\Omega_{mm} = \triangle^{1/2}_{m(m_1,n_1)}\triangle^{1/2}_{m(m_2,n_2)}$ as before with $\triangle_{e(m_j,n_j)}$ and $\triangle_{m(m_j,n_j)}$ defined in (3.11) and (3.13), respectively. One can check easily that only for the (m, m) case above, $U(m_1,n_1;m_2,n_2)$ can vanish and this occurs at $|m_1|/n_1 = |m_2|/n_2$ ($n_1 n_2 > 0$) or $f_1 = \pm f_2$, giving a vanishing amplitude, an indication of preservation of certain number of spacetime supersymmetries. This is interesting and a bit counterintuitive, and it indicates that when the magnetic flux in one non-threshold bound state shares no common direction with the magnetic flux in the other non-threshold bound state and when their magnitude is the same, then the force acting between the two cancels. As will be shown in the appendix, the above condition is precisely the one for this system to preserve also 1/4 of spacetime supersymmetries.

Following what we did in the previous subsection, the large separation amplitude can be obtained from (3.58) and is

$$\Gamma = \frac{C(m_1,n_1;m_2,n_2)}{Y^{7-p}}, \quad (3.60)$$

where

$$C(m_1,n_1;m_2,n_2) = \frac{c_p^2\, V_{p+1}\, U(m_1,n_1;m_2,n_2)}{(7-p)\Omega_{8-p}}, \quad (3.61)$$

with $U(m_1,n_1;m_2,n_2)$ now given in (3.59).

Note that if the condition $|m_1|/n_1 = |m_2|/n_2$ ($n_1 n_2 > 0$), giving rise to $U(m_1,n_1;m_2,n_2) = 0$, for the case of (m, m) mentioned above is excluded, we have then $U(m_1,n_1;m_2,n_2) > 0$ for all cases considered in this subsection. Note also the following factor in the numerator in the infinite product in the integrand in the second equality in (3.58)

$$\prod_{j=1}^{2}(1-e^{\pi i(\nu_1+(-)^j\nu_2)}|z|^{2n})^2(1-e^{-\pi i(\nu_1+(-)^j\nu_2)}|z|^{2n})^2$$
$$=\left[(1+|z|^{4n})^2+2|z|^{4n}(\cos 2\pi\nu_1+\cos 2\pi\nu_2)-4|z|^{2n}(1+|z|^{4n})\cos\pi\nu_1\cos\pi\nu_2\right]^2>0, \quad (3.62)$$

so the sign of the amplitude is again determined by the following factor in the denominator in the infinite product in the integrand

$$\prod_{j=1}^{2}(1-e^{2\pi i\nu_j}|z|^{2n})(1-e^{-2\pi i\nu_j}|z|^{2n})$$
$$=(1-2|z|^{2n}\cos 2\pi\nu_1+|z|^{4n})(1-2|z|^{2n}\cos 2\pi\nu_2+|z|^{4n}), \quad (3.63)$$

which is positive for the case of (m, m) for which both ν_1 and ν_2 are real and for large t for the remaining case but can be negative for small t for this case since now ν_1 is imaginary. Therefore the interaction amplitude is positive for the case of (m, m) once again as expected, reflecting the attractive nature of the interaction between BS1 and BS2 in the present case. For the case of (e, m) while the large separation amplitude is still positive and the corresponding interaction is attractive, the small separation amplitude is once again uncertain for the same reason mentioned in the previous subsection in a similar situation. We again expect interesting physics to arise in the small t limit for these two cases to which we will turn next.

The best picture to study the small t physics is in terms of open string description [29] and this can be realized by the transformation of integration variable $t \to t' = 1/t$ which converts the closed string cylinder diagram to the open string annulus diagram. So in terms of the annulus variable t', with (3.39), the amplitude (3.58) is now

$$\Gamma = -\frac{4\,n_1 n_2\,V_{p+1}\,\tan\pi\nu_1\,\tan\pi\nu_2}{(8\pi^2\alpha')^{\frac{1+p}{2}}}\int_0^\infty\frac{dt'}{t'}\,e^{-\frac{Y^2t'}{2\pi\alpha'}}\,t'^{\frac{3-p}{2}}\,\frac{\theta_1^2(-i\frac{\nu_1-\nu_2}{2}t'|it')\,\theta_1^2(-i\frac{\nu_1+\nu_2}{2}t'|it')}{\eta^6(it')\,\theta_1(-i\nu_1 t'|it')\theta_1(-i\nu_2 t'|it')},$$
$$= -\frac{2^4\,n_1 n_2\,V_{p+1}\,\tan\pi\nu_1\,\tan\pi\nu_2}{(8\pi^2\alpha')^{\frac{1+p}{2}}}\int_0^\infty\frac{dt'}{t'}\,e^{-\frac{Y^2t'}{2\pi\alpha'}}\,t'^{\frac{3-p}{2}}\,\frac{\sin^2(-i\pi\frac{\nu_1-\nu_2}{2}t')\sin^2(-i\pi\frac{\nu_1+\nu_2}{2}t')}{\sin(-i\pi\nu_1 t')\sin(-i\pi\nu_2 t')}$$
$$\times\prod_{n=1}^{\infty}\frac{1}{(1-|z|^{2n})^4}\prod_{j=1}^{2}\frac{(1-e^{\pi(\nu_1+(-)^j\nu_2)t'}|z|^{2n})^2(1-e^{-\pi(\nu_1+(-)^j\nu_2)t'}|z|^{2n})^2}{(1-e^{2\pi\nu_j t'}|z|^{2n})(1-e^{-2\pi\nu_j t'}|z|^{2n})}, \quad (3.64)$$

where $|z| = e^{-\pi t'}$. We once again follow [1, 26] to discuss the analytic structure of the above amplitude and the associated physics. For the present (m, m) case, both ν_1 and ν_2 are real with their respective range $0 < \nu_1, \nu_2 < 1/2$ and the above amplitude appears positive and has no simple poles on the positive t'-axis. For the same reason mentioned in the previous subsection, this amplitude has no imaginary part, therefore giving zero rate of open string pair production as expected. Note that this amplitude has also a singularity as $t' \to \infty$ when $Y \leq \pi\sqrt{2|\nu_1-\nu_2|\alpha'}$, i.e., on the order of string scale, and this happens also in a similar fashion as in the brane/antibrane system mentioned in previous subsection but now caused purely by the presence of magnetic fluxes. This singularity can be examined from

$$\lim_{t'\to\infty}\frac{e^{-\frac{Y^2t'}{2\pi\alpha'}}\theta_1^2(\frac{\nu_1-\nu_2}{2i}t'|it')\,\theta_1^2(\frac{\nu_1+\nu_2}{2i}t'|it')}{-\eta^6(it')\,\theta_1(-i\nu_1t'|it')\theta_1(-i\nu_2t'|it')}\sim\lim_{t'\to\infty}\frac{e^{-\frac{Y^2t'}{2\pi\alpha'}}\sin^2(\pi\frac{\nu_1-\nu_2}{2i}t')\sin^2(\pi\frac{\nu_1+\nu_2}{2i}t')}{i^2\sin(-i\pi\nu_1t')\sin(-i\pi\nu_2t')},$$
$$\sim\lim_{t'\to\infty}e^{-\frac{Y^2t'}{2\pi\alpha'}}[e^{\pi|\nu_1-\nu_2|t'}+\mathcal{O}(1)].\tag{3.65}$$

The appearance of the divergent amplitude also indicates the breakdown of the calculations, signalling the onset of tachyonic instability caused by the magnetic fluxes and the relaxation of the system to form a new non-threshold bound state. In addition, that the open string tachyon mode appears to arise is also indicated from the leading term $e^{\pi|\nu_1-\nu_2|t'}$, which diverges in the short cylinder limit $t'\to\infty$, in the expansion of the θ-functions and η-function in (3.64) in the open string channel. This is supported further by the evidence that this divergence, therefore the tachyon mode, disappears when $|\nu_1-\nu_2|$ vanishes but when this happens the amplitude also vanishes, indicating the underlying system being BPS and preserving certain number of spacetime supersymmetries as mentioned earlier. Once again, however, the detail of the underlying dynamical process requires further understanding.

For the remaining case, $\nu_1=i\nu_{10}$ is imaginary with $0<\nu_{01}<\infty$ and $\nu_2=\nu_{20}$ is real with $0<\nu_{20}<1/2$. Then from the second expression in (3.64), we have the amplitude

$$\Gamma=\frac{4\,n_1n_2\,V_{p+1}\,\tanh\pi\nu_{10}\,\tan\pi\nu_{20}}{(8\pi^2\alpha')^{\frac{1+p}{2}}}\int_0^\infty\frac{dt'}{t'}\,e^{-\frac{Y^2t'}{2\pi\alpha'}}\,t'^{\frac{3-p}{2}}\frac{(\cos\pi\nu_{10}t'-\cosh\pi\nu_{20}t')^2}{\sin(\pi\nu_{10}t')\sinh(\pi\nu_{20}t')}$$
$$\times\prod_{n=1}^{\infty}\frac{\prod_{j=1}^2(1-e^{\pi(i\nu_{10}+(-)^j\nu_{20})t'}|z|^{2n})^2(1-e^{-\pi(i\nu_{10}+(-)^j\nu_{20})t'}|z|^{2n})^2}{(1-|z|^{2n})^4(1-2|z|^{2n}\cos2\pi\nu_{10}t'+|z|^{4n})(1-e^{2\pi\nu_{20}t'}|z|^{2n})(1-e^{-2\pi\nu_{20}t'}|z|^{2n})},\tag{3.66}$$

where the following factor is positive and can be expressed as

$$\prod_{j=1}^2(1-e^{\pi(i\nu_{10}+(-)^j\nu_{20})t'}|z|^{2n})^2(1-e^{-\pi(i\nu_{10}+(-)^j\nu_{20})t'}|z|^{2n})^2$$
$$=[(1+|z|^{4n})(1+|z|^{4n}-4|z|^{2n}\cos\pi\nu_{10}t\cosh\pi\nu_{20}t')+2|z|^{4n}(\cos2\pi\nu_{10}t'+\cosh2\pi\nu_{20}t')]^2.\tag{3.67}$$

When $\nu_{20}\neq0$ as we always assume in this subsection, the above amplitude has simple poles occurring at $t'_k=k/\nu_{10}$ with $k=1,2,\ldots$ and the number of simple poles in the present case doubles in comparison with the case when the two fluxes share at least one common direction as discussed in the previous subsection and in [1, 26, 30, 32–34, 36].[7] Then the rate of pair production of open strings per unit worldvolume is the imaginary part of the above

[7] In some of these papers, the number of simple poles appeared also given by $t'_k=k/\nu_{01}$ with $k=1,2,\ldots$ due to that their amplitude expressions were not simplified using the identity (3.30) for various θ-functions and the contribution to the amplitude from each even k is actually zero. This can also be seen easily from (3.66) when taking $\nu_{20}=0$.

amplitude, which is the sum of the residues of the poles times π following the prescription given in the previous subsection as described in [30, 33]. We then have the rate as

$$\begin{aligned}\mathcal{W} &\equiv -\frac{2\mathrm{Im}\Gamma}{V_{p+1}},\\ &= \frac{8n_1n_2\tanh\pi\nu_{10}\tan\pi\nu_{20}}{\nu_{10}}\sum_{k=1}^{\infty}(-)^{k+1}\left(\frac{\nu_{10}}{8k\pi^2\alpha'}\right)^{\frac{1+p}{2}} e^{-\frac{kY^2}{2\pi\nu_{10}\alpha'}}\frac{\left[\cosh\frac{k\pi\nu_{20}}{\nu_{10}}-(-)^k\right]^2}{\frac{\nu_{10}}{k}\sinh\frac{k\pi\nu_{20}}{\nu_{10}}}\\ &\quad\times\prod_{n=1}^{\infty}\frac{\left[1-2(-)^k e^{-\frac{2nk\pi}{\nu_{10}}}\cosh\frac{k\pi\nu_{20}}{\nu_{10}}+e^{-\frac{4nk\pi}{\nu_{10}}}\right]^4}{\left[1-e^{-\frac{2nk\pi}{\nu_{10}}}\right]^6\left[1-e^{-\frac{2k\pi}{\nu_{10}}(n-\nu_{20})}\right]\left[1-e^{-\frac{2k\pi}{\nu_{10}}(n+\nu_{20})}\right]},\end{aligned} \tag{3.68}$$

which reduces to the rate (3.43) given in the previous subsection when we set $\nu_{20}\to 0$ and $\nu_{10}=\nu_0$ in the above as expected. The rate is highly suppressed by the separation and the integer k and for each given k the corresponding term appears likely enhanced by both ν_{10} and ν_{20}. The latter is particularly evident for large magnetic flux for which $\nu_{20}\to 1/2$ and the front factor $\tan\pi\nu_{20}\to\infty$. Note that the odd k gives positive contribution while the even k gives negative contribution to the above rate. Also $k=1$ term gives the leading positive contribution to the rate.[8] All these indicate that the presence of magnetic flux appears to enhance the rate. Let us consider the small electric flux case. For this, we need to consider only the leading term which is given by the $k=1$ term in the above and for a fixed non-vanishing ν_{20} it is

$$\mathcal{W}\approx\frac{4n_1n_2\pi}{\nu_{10}}\left(\frac{\nu_{10}}{8\pi^2\alpha'}\right)^{\frac{1+p}{2}} e^{-\frac{Y^2}{2\pi\nu_{10}\alpha'}}e^{\frac{\pi\nu_{20}}{\nu_{10}}}\tan\pi\nu_{20}, \tag{3.69}$$

which is greatly enhanced by a factor of $e^{\frac{\pi\nu_{20}}{\nu_{10}}}\tan\pi\nu_{20}/(8\nu_{10})$ in comparison with the similar rate given in the previous subsection. In particular, when the separation is on the order of $\pi\sqrt{2\nu_{20}\alpha'}$, i.e., the string scale, this rate can become very significantly large. In other words, when the two bound states are in a close contact, the open string pair production can be very significant. When this happens, we need to use the following better approximated rate to make the evaluation

$$\mathcal{W}=\frac{4n_1n_2\,\pi\,\tan\pi\nu_{20}}{(8\pi^2\alpha')^{\frac{p+1}{2}}}\sum_{k=1}^{\infty}(-)^{k+1}\left(\frac{\nu_{10}}{k}\right)^{\frac{p-1}{2}} e^{-\frac{k}{2\pi\nu_{10}\alpha'}\left(Y^2-2\pi^2\nu_{20}\alpha'\right)}. \tag{3.70}$$

[8] A different enhancement of a similar rate by a magnetic flux in a different context, i.e., purely bosonic string case, was explicitly considered in [37] (The corresponding supersymmetric case was briefly mentioned). In this case, what has been considered is an open string placed in an electric-magnetic background and the two ends of an open string experience the same flux which can have both electric and magnetic components, a generalization of the bosonic case considered in [30] by including a magnetic component. Our consideration here is completely different: we have a system of two branes with a separation. In terms of open string description, one end of string experiences an electric flux living on one stack of D-branes while its other end experiences a magnetic flux living on the other stack of D-branes placed parallel at a separation, a superstring analysis. As a result, unlike the case in [37], the present rate has a dependence on the brane separation. Moreover, the enhancement factor in [37] is merely a Born-Infeld factor $\sqrt{1+f_2^2}$ (expressed in our notations), independent of the electric component, while the present enhancement is given as $e^{\frac{\pi\nu_{20}}{\nu_{10}}}|f_2|/(8\nu_{10})$ for fixed f_2 with $\tan\pi\nu_{20}=|f_2|$, completely different. The ratio of these two is $e^{\frac{\pi\nu_{20}}{\nu_{10}}}|f_2|/(8\nu_{10}\sqrt{1+f_2^2})\approx e^{\frac{\pi\nu_{20}}{\nu_{10}}}/(8\nu_{10})\gg 1$ for $|f_2|\geq 1$ and small ν_{10}.

Let us make some numerical estimation of the rate given in (3.69) when the approximation is valid and this may serve for sensing the significance of the rate mentioned above. For this purpose, we take $\nu_{20} = 2/5$, $\nu_{10} = 1/50$ and the enhance factor given above is then

$$e^{\frac{\pi\nu_{20}}{\nu_{10}}} \frac{\tan \pi\nu_{20}}{8\,\nu_{10}} = e^{20\pi} \frac{25 \tan 0.4\pi}{4} \sim 3.6 \times 10^{28}. \tag{3.71}$$

We also calculate the rate in string units for a few sample cases (note that we need to have $p \geq 3$ as mentioned earlier) in the following as

$$\begin{aligned} (2\pi\alpha')^{\frac{p+1}{2}} \mathcal{W} &\approx n_1 n_2 \left(\frac{\nu_{10}}{4\pi}\right)^{\frac{p-1}{2}} e^{-\frac{Y^2 - 2\pi^2\nu_{20}\alpha'}{2\pi\nu_{10}\alpha'}} \tan \pi\nu_{20} \\ &\approx n_1 n_2 \left(\frac{\nu_{10}}{4\pi}\right)^{\frac{p-1}{2}} \tan \pi\nu_{20}, \\ &\approx \begin{cases} 0.49 \ \text{for}\, p = 3, \\ 0.03 \ \text{for}\, p = 4, \end{cases} \end{aligned} \tag{3.72}$$

where we have taken $Y = \pi\sqrt{2\nu_{20}\alpha'} + 0^+ \approx 2.81\sqrt{\alpha'}$, $n_1 = 10$ and $n_2 = 10$. So the rate can indeed be significant for $p = 3, 4$ and can be larger if we take larger n_1 and n_2 when the brane separation is a few times of string scale and before the tachyon condensation starts to function. The above enhancement may have potentially realistic applications, for example, to objects carrying both electric and magnetic fluxes or to one object carrying an electric flux and the other carrying a magnetic flux in a similar situation in our Universe at early times or to the present macroscopic objects carrying similar fluxes for which n_1 and n_2 are very large, if string theories are indeed relevant to our real world. If this indeed happens, the produced large number of open string pairs can in turn annihilate to give highly concentrated high energy photons, for example, which may have observational consequence such as the Gamma-ray burst. The related effects may also serve as an indication for the existence of extra dimensions since it requires $p \geq 3$ and the transverse dimensions are also necessary. Moreover, the rate for $p = 3$ is at least one-order of magnitude larger than the other $p \geq 4$, the underlying dynamics may select 4 spacetime dimensions as special against the others, if a brane-world view is taken. This enhancement will not occur if the electric flux points along either of the two spatial directions of the magnetic flux as our results given in the previous subsection show. Note that for small but fixed electric flux as given in (3.69) or even for a finite electric flux as in (3.68), the corresponding rate diverges when the magnetic flux becomes large and this may indicate a new instability to occur.

There is another singularity which can be examined by looking at the integrand of (3.66) at large t' when $Y - \pi\sqrt{2\nu_{20}\alpha'} \to 0^-$ as

$$\lim_{t' \to \infty} \frac{e^{-\frac{Y^2 t'}{2\pi\alpha'}} (\cos \pi\nu_{10}t' - \cosh \pi\nu_{20}t')^2}{\sinh(\pi\nu_{20}t')} \sim \lim_{t' \to \infty} e^{-\frac{t'}{2\pi\alpha'}(Y^2 - 2\pi^2\nu_{20}\alpha')}, \tag{3.73}$$

which signals also the onset of tachyonic instability as in the pure magnetic case (Note that this does not require a weak electric flux and is associated with the real part of the amplitude). For strong electric flux, each term in (3.68) always diverges. So when $y >$

$\pi\sqrt{2\nu_{20}\alpha'}$, the pair production of open strings is the only process to lower the system energy but as $Y \to \pi\sqrt{2\nu_{20}\alpha'}$, the tachyonic instability starts to occur and the pair production continues and and become larger and larger. So the dynamics here may be rich and needs further study before we can be certain to which the final state of this system leads.

4 Summary

In this paper, we exhaust the amplitude calculations of [1] between two non-threshold bound states of the type of either (F, D_p) or $(D_{(p-2)}, D_p)$ or both for the remaining cases as specified in the Introduction. We find that the amplitude has the same basic structure when the two fluxes share at least one common index. The amplitude is more general and includes the previous one as a special case when the two fluxes share no common index. The nature of the force acting between two bound states is always attractive when the two fluxes are both magnetic or magnetic dominant in a sense defined in the text. For the rest of cases considered in this paper, we are certain that the interaction is attractive only at large separation. We also find that only for two situations the interaction amplitude can vanish and if this happens, the underlying system preserves 1/4 of spacetime supersymmetries. One is that the two fluxes have different nature with the magnetic flux sharing one common index with the electric one, related to each other by (3.28), and the other is when the two fluxes are both magnetic sharing no common index and having the same magnitude.

We also study the analytic structures of various amplitudes considered in this paper. When the two fluxes are both magnetic or when the magnetic flux dominates over the electric flux in effect and shares one common index, the amplitude diverges when the brane separation is on the order of string scale just like the brane/antibrane situation studied in [39, 40], signalling the onset of tachyonic instability, and this may serve as the dynamical process to lower the energy of the system and to relax it to form the final stable bound state as indicated in [38]. For the rest of cases studied, i.e., the cases with at least one or one dominant electric flux present, there is always a non-vanishing open string pair production rate associated with this flux. In particular, this rate can be greatly enhanced when there is in addition a magnetic flux present and the electric flux is weak but with the two sharing no common index. These may have realistic physical consequences for and potential applications to objects in our Universe and their evolution when they carry a weak electric field and a reasonable but fixed magnetic field and if string theories are relevant to our real world. If this happens indeed, it may serve also as an indication for the existence of extra dimensions since the spatial dimensionality of the D_p branes has now to be $p \geq 3$. Further, our usual 4 dimensional spacetime seemly has also a special role since the rate in this case is at least one-order of magnitude larger than the other relevant cases. Pursuing all these applications is beyond the scope of the present work and will be postponed to future projects.

In addition to the usual strong electric flux singularity for the pair production rate, for the present case we also find two new singularities when the brane separation is on the order of string scale: one is associated with the pair production rate for a weak electric flux with a large magnetic flux and the other is from the real part of the amplitude and associated with the onset of tachyonic instability but independent of the electric flux requirement. The dynamics here may be rich and needs further study before we can be sure about the final state of the system.

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A Supersymmetry

In this appendix, we will confirm explicitly the preservation of 1/4 spacetime supersymmetries for each of the three cases mentioned in the text when the corresponding amplitude vanishes, namely, (e, m) when the electric flux shares a common spatial direction with the magnetic flux along with (3.28) satisfied, and the (m, m) case when the two magnetic fluxes share no common direction along with .

For this, let us first note that the condition for 1/2-supersymmetry preservation for each bound state is

$$\epsilon_1 = \eta\, \Gamma^{p+1} \cdots \Gamma^9 U(\hat{F}_j)\, \epsilon_2, \tag{A.1}$$

where ϵ_1 and ϵ_2 are the two Majorana-Weyl supersymmetry parameters in IIA/IIB string theory (the two have the opposite chirality in IIA but the same chirality in IIB), $U(\hat{F}_j)$ is defined in (2.12) due to the presence of flux, the sign $\eta = \pm$ and $j = 1, 2$. This condition reduces to the familiar one when the flux is set to zero (note that we have used $\Gamma^0\Gamma^1 \cdots \Gamma^p = \pm\Gamma^{p+1} \cdots \Gamma^9\Gamma^{11}$ and $\Gamma^{11}\epsilon_2 = \pm\epsilon_2$). The above clearly indicates that the SUSY parameter ϵ_1 is completely determined by ϵ_2, therefore only half supersymmetry is preserved for a given bound state when it is isolated, the well-known fact. For the cases under consideration, we have two bound states with the Dp branes in one bound state placed parallel to those in the other bound state at a separation. Therefore, for such a system to preserve certain number of supersymmetries, we need to have (A.1) hold simultaneously for $j = 1, 2$. This is equivalent to having (A.1) hold for either $j = 1$ or $j = 2$ plus the following

$$U(\hat{F}_1)\, \epsilon_2 = U(\hat{F}_2))\, \epsilon_2 \tag{A.2}$$

for non-vanishing ϵ_2. The number of non-vanishing components of ϵ_2 satisfying the above equation determines the number of unbroken SUSY for such a system.

Let us check the (e, m) case first. For this case,

$$U(\hat{F}_1) = \frac{1 + f_1\Gamma^0\Gamma^1}{\sqrt{1 - f_1^2}}, \tag{A.3}$$

where we choose the electric flux along x^1-direction. Without loss of generality we can choose the magnetic flux along x^1 and x^2 directions and then

$$U(\hat{F}_2) = \frac{1 + f_2\Gamma^1\Gamma^2}{\sqrt{1 + f_2^2}}. \tag{A.4}$$

So (A.2) now becomes

$$(1 \pm \mathcal{B})\, \epsilon_2 = 0, \tag{A.5}$$

where we have expressed f_2 in terms of f_1 using (3.28) and

$$\mathcal{B} = \sqrt{1 - f_1^2}\Gamma^0\Gamma^2 \pm f_1\Gamma^0\Gamma^1. \tag{A.6}$$

Since $\mathrm{Tr}\,\mathcal{B} = 0$ and $\mathcal{B}^2 = I_{32\times 32}$ with $I_{32\times 32}$ the unit matrix, (A.5) says that only half of the components of ϵ_2 can be non-vanishing and therefore overall only 1/4 of the spacetime supersymmetries can be preserved by this configuration. The (m, e) case can be similarly discussed and the conclusion remains the same, i.e., the underlying system preserves only 1/4 of overall spacetime supersymmetries.

We now move to the (m, m) case mentioned above. For this case, we choose the first magnetic flux $\hat{F}_1$ along x^1 and x^2 directions and so

$$U(\hat{F}_1) = \frac{1 + f_1\Gamma^1\Gamma^2}{\sqrt{1 + f_1^2}}, \tag{A.7}$$

and without loss of generality we choose the second magnetic flux $\hat{F}_2$ along x^3 and x^4 directions and so

$$U(\hat{F}_2) = \frac{1 + f_2\Gamma^3\Gamma^4}{\sqrt{1 + f_2^2}}. \tag{A.8}$$

(A.2) now reduces to

$$(1 \pm \Gamma^1\Gamma^2\Gamma^3\Gamma^4)\, \epsilon_2 = 0, \tag{A.9}$$

where we have used $f_1 = \pm f_2$ which is precisely the one giving the vanishing amplitude in subsection 3.2. Since $\mathrm{Tr}\,\Gamma^1\Gamma^2\Gamma^3\Gamma^4 = 0$ and $(\Gamma^1\Gamma^2\Gamma^3\Gamma^4)^2 = I_{32\times 32}$, by the same token, this system preserves also 1/4 of total spacetime supersymmetries.

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